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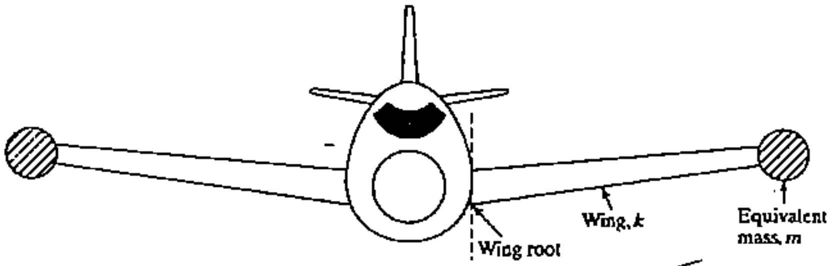
MEN 330
EXAM 2
SPRING 2005

100
Very good

1) (15 pts) An airplane, taxiing on a runway, encounters a bump. As a result, the root of the wing is subjected to a displacement that is expressed as

$$y(t) = \begin{cases} Y\left(\dfrac{t^2}{t_0^{\,2}}\right), & 0 \le t \le t_0 \\ 0, & t > t_0 \end{cases}$$

Assume that the wing has a stiffness k and set-up the Duhamel integral(s) which would calculate the response of the wing tip mass m for $t \in [0, \infty[$. Set-up the integral(s) fully but **do not** attempt to solve the problem.



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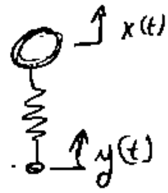


for $0 \le t < t_0$

$F(t) = K\,y(t)$

$$x(t) = \frac{1}{m\omega_d} \int_0^t F(\tau) \sin \omega_d (t-\tau)\, d\tau$$

with $\zeta = 0 \Rightarrow \omega_d = \omega_n$ $\qquad \omega_n = \sqrt{\dfrac{K}{m}}$

$$\Rightarrow \boxed{x(t) = \frac{1}{m\omega_n} \int_0^t \frac{KY}{t_0^2}\,\tau^2 \sin\omega_n(t-\tau)\, d\tau} \text{ for } 0 \le t < t_0$$

✓

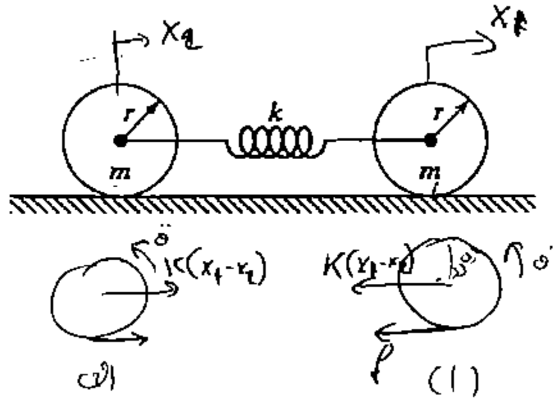
for $t > t_0$

$$x(t) = \frac{1}{m\omega_n} \int_0^{t_0} \frac{KY}{t_0^2}\,\tau^2 \sin\omega_n(t-\tau)\, d\tau + \frac{1}{m\omega_n}\int_{t_0}^{t} 0 \,\sin\omega_n(t-\tau)\, d\tau$$

$$\Rightarrow \boxed{\text{for } t > t_0 \quad x(t) = \frac{1}{m\omega_n} \int_0^{t_0} \frac{KY}{t_0^2}\,\tau^2 \sin\omega_n(t-\tau)\, d\tau}$$

✓

2) (15 pts) Two identical circular cylinders of radius r and mass m each, are connected by a spring of stiffness k. Determine the natural frequencies of oscillation under no-slip conditions.



$\Sigma M_{(1)} = \; f r = I\ddot{\theta} \Rightarrow f r = \frac{1}{2} m r^2 \ddot{\theta}$ let $r\ddot{\theta} = \ddot{x}$

$\Rightarrow f r = \frac{1}{2} m r \ddot{x}_1 \Rightarrow f = \frac{1}{2} m \ddot{x}_1$

15

$\Sigma F = -K(x_1 - x_2) - f - m\ddot{x}_1 = 0$

$\Rightarrow m\ddot{x}_1 + \frac{1}{2} m \ddot{x}_1 + Kx_1 - Kx_2 = 0$

$\frac{3}{2} m \ddot{x}_1 + Kx_1 - Kx_2$ ——— (1)

$\Sigma M_{(2)} = \; f = -\frac{1}{2} m \ddot{x}_2$

$\Sigma F_{(2)} = \; K(x_1 - x_2) + f - m\ddot{x}_2 = 0$

$\Rightarrow m\ddot{x}_2 + \frac{1}{2} m \ddot{x}_2 - Kx_1 + Kx_2 = 0$

$\Rightarrow \frac{3}{2} m \ddot{x}_2 - Kx_1 + Kx_2$ ——— (2)

$$\begin{pmatrix} \frac{3}{2}m & 0 \\ 0 & \frac{3}{2}m \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} K & -K \\ -K & K \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

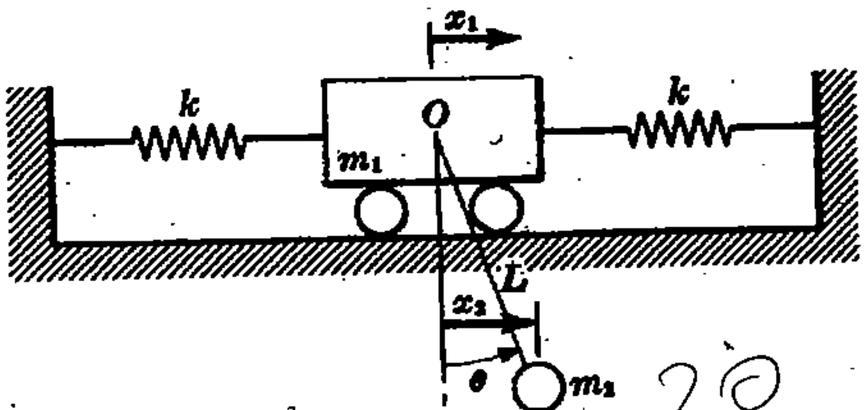
$$\det(-\omega^2 M + K) = 0 \Rightarrow \begin{vmatrix} -\frac{3}{2} m\omega^2 + K & -K \\ -K & -\frac{3}{2} m\omega^2 + K \end{vmatrix} = 0$$

$\Rightarrow \frac{9}{4} m^2 \omega^4 - 3mK\omega^2 + K^2 - K^2 = 0$

$\Rightarrow \frac{9}{4} m^2 \omega^4 - 3mK\omega^2 = 0 \Rightarrow \omega^2\left(\frac{9}{4} m^2\omega^2 - 3mK\right) = 0$

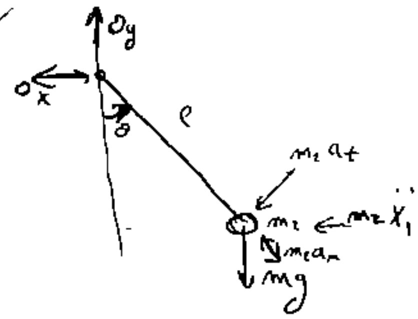
$\Rightarrow \boxed{\omega = 0}$ $\omega_2^2 = \frac{3mK}{\frac{9}{4}m^2} \Rightarrow \boxed{\omega_2 = \sqrt{\frac{4K}{3m}}}$

3) (20 pts) Determine the natural frequencies of the system for small oscillations.



FBD 1.

$\Sigma M_o = I_o \ddot{\theta}$ where $I_o = m_2 \ell^2$



$\Rightarrow -m_2 g \ell \theta - m_2 \ell \ddot{x}_1 - m_2 L^2 \ddot{\theta} = 0$

$\Rightarrow m_2 L^2 \ddot{\theta} + m_2 L \ddot{x}_1 + m_2 g L \theta = 0$ — ①

$\Sigma F_{(x)} \Rightarrow -O_x - m_2 \ddot{x}_1 - m_2 a_t - m_2 a_n \theta = 0$

$\Rightarrow -O_x - m_2 \ddot{x} - m_2 \ell \ddot{\theta} + m_2 L \dot{\theta}^2 \theta = 0$ $\quad a_n = \frac{V^2}{\ell} = \frac{\ell^2 \dot{\theta}^2}{\ell} = L\dot{\theta}^2$

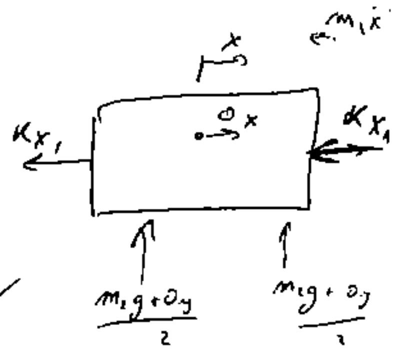
$\Rightarrow O_x = m_2 L \dot{\theta}^2 \theta - m_2 \ddot{x} - m_2 \ell \ddot{\theta}$

FBD 2

$\Sigma F_x = O_x - 2Kx_1 - m_1 \ddot{x} = 0$

can be neglected

$m_2 L \dot{\theta}^2 \theta - m_2 \ddot{x}_1 - m_2 \ell \ddot{\theta} - 2Kx_1 - m_1 \ddot{x}_1 = 0.$



$\Rightarrow m_1 \ddot{x}_1 + m_2 \ddot{x}_1 + m_2 \ell \ddot{\theta} + 2Kx = 0$

$\Rightarrow (m_1 + m_2)\ddot{x} + m_2 \ell \ddot{\theta} + 2Kx.$ — ②

$$\underbrace{\begin{pmatrix} m_1+m_2 & m_2 L \\ m_2 L & m_2 L^2 \end{pmatrix}}_{M} \underbrace{\begin{pmatrix} \ddot{x} \\ \ddot{\theta} \end{pmatrix}}_{\ddot{x}} + \underbrace{\begin{pmatrix} 2K & 0 \\ 0 & m_2 g L \end{pmatrix}}_{K} \underbrace{\begin{pmatrix} x \\ \theta \end{pmatrix}}_{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

~~det M~~

$$\det(-\omega^2 M + K) = 0 \Rightarrow \begin{vmatrix} -(m_1+m_2)\omega^2 + 2K & -m_2 L\omega^2 \\ -m_2 L\omega^2 & -m_2 L^2\omega^2 + m_2 g L \end{vmatrix} = 0.$$

$$(m_1+m_2)m_2 L^2\omega^4 - (m_1+m_2)m_2 g L\omega^2 - 2m_2 L^2 K\omega^2 + 2m_2 g L K - m_2^2 L^2\omega^4 = 0.$$

Solve for ω

$$\Rightarrow m_1 \cancel{m_2} L^2\omega^4 + m_2^2 L^2\omega^4 - m_1 \cancel{m_2} g L\omega^2 - m_2^2 g \cancel{L}\omega^2 - 2\cancel{m_2} L^2 K\omega^2 + 2 m_2 g \cancel{L} K - m_2^2 \cancel{L^2\omega^4} = 0$$

$$m_1 L\omega^4 - m_1 g\omega^2 - m_2 g\omega^2 - 2LK\omega^2 + 2gK = 0 \quad —(3)$$

$$\underbrace{m_1 L}_{a}\omega^4 - \underbrace{(m_1 g + m_2 g + 2LK)}_{b}\omega^2 + \underbrace{2gK}_{c} = 0.$$

The natural frequencies ω_1 and ω_2 are the roots of eq. (3).

$$\Delta = b^2 - 4ac =$$

$$\omega_1 = \frac{b + \sqrt{\Delta}}{2a} \qquad \omega_2 = \frac{b - \sqrt{\Delta}}{2a}$$